[tex111] Density of energy levels for ideal quantum gas in \mathcal{D} dimensions

(a) Consider a nonrelativistic ideal quantum gas in \mathcal{D} dimensions and confined to a box of volume $V = L^{\mathcal{D}}$ with rigid walls. The energy-momentum relation is $\epsilon = \hbar^2 |\mathbf{k}|^2 / 2m$ and the density of states in reciprocal space is $\rho(\mathbf{k}) = (L/2\pi)^{\mathcal{D}} = \text{const.}$ Show that the density of energy levels is

$$D(\epsilon) = \frac{V}{\Gamma(\mathcal{D}/2)} \left(\frac{2\pi m}{h^2}\right)^{\mathcal{D}/2} \epsilon^{\mathcal{D}/2-1}.$$

(b) Consider an ultrarelativistic ideal quantum gas in \mathcal{D} dimensions and confined to a box of volume $V = L^{\mathcal{D}}$ with rigid walls. The energy-momentum relation is $\epsilon = \hbar |\mathbf{k}| c$ and the density of states in reciprocal space is $\rho(\mathbf{k}) = (L/2\pi)^{\mathcal{D}} = \text{const.}$ Show that the density of energy levels is

$$D(\epsilon) = \frac{2V\pi^{\mathcal{D}/2}\epsilon^{\mathcal{D}-1}}{\Gamma(\mathcal{D}/2)(ch)^{\mathcal{D}}}.$$

Solution: