Ideal Bose-Einstein gas: isobars [tsl48]

A phase transition at $T_c > 0$ takes place in all dimensions $\mathcal{D} \geq 1$. However, the existence of a BEC requires $v_c > 0$, which is realized only for $\mathcal{D} > 2$.

Isobar at $T > T_c$:

$$\frac{v}{v_p} = \frac{\left[g_{\mathcal{D}/2+1}(z)\right]^{\mathcal{D}/(\mathcal{D}+2)}}{g_{\mathcal{D}/2}(z)}, \qquad \frac{T}{T_p} = \left[g_{\mathcal{D}/2+1}(z)\right]^{-2/(\mathcal{D}+2)}.$$

Critical point:

$$\frac{v_c}{v_p} = \frac{\left[\zeta(\mathcal{D}/2+1)\right]^{\mathcal{D}/(\mathcal{D}+2)}}{\zeta(\mathcal{D}/2)} = \begin{cases} 0 & \mathcal{D}=1 \\ 0 & \mathcal{D}=2 \\ 0.383 & \mathcal{D}=3 \\ 1 & \mathcal{D}=\infty \end{cases}$$

$$\frac{T_c}{T_p} = \left[\zeta(\mathcal{D}/2+1)\right]^{-2/(\mathcal{D}+2)} = \begin{cases} 0.527 & \mathcal{D}=1 \\ 0.779 & \mathcal{D}=2 \\ 0.884 & \mathcal{D}=3 \\ 1 & \mathcal{D}=\infty \end{cases}$$