Bose-Einstein condensation [tsl38]

Particles in the gas phase and in the Bose-Einstein condensate (BEC):

$$\mathcal{N} = \frac{V}{\lambda_T^{\mathcal{D}}} g_{\mathcal{D}/2}(z) + \frac{z}{1-z} = \mathcal{N}_{gas} + \mathcal{N}_{BEC}.$$

Consider process at $v = \text{const}$. Onset of macroscopic population of the lowest energy level begins when the fugacity locks in to the value $z = 1$. This scenario is realized in $\mathcal{D} > 2$.

$$\frac{z}{1-z} = \begin{cases} \mathrm{O}(1), & z < 1, \\ \mathrm{O}(\mathcal{N}), & z = 1. \end{cases}$$

$$T \geq T_c: \quad \frac{\mathcal{N}_{gas}}{\mathcal{N}} = 1, \quad \frac{\mathcal{N}_{BEC}}{\mathcal{N}} = 0.$$

$$T \leq T_c: \quad \begin{cases} \dfrac{\mathcal{N}_{gas}}{\mathcal{N}} = \dfrac{[V/\lambda_T^{\mathcal{D}}]\zeta(\mathcal{D}/2)}{[V/\lambda_{T_c}^{\mathcal{D}}]\zeta(\mathcal{D}/2)} = \left(\dfrac{T}{T_c}\right)^{\mathcal{D}/2}, \\ \dfrac{\mathcal{N}_{BEC}}{\mathcal{N}} = 1 - \dfrac{\mathcal{N}_{gas}}{\mathcal{N}} = 1 - \left(\dfrac{T}{T_c}\right)^{\mathcal{D}/2}. \end{cases}$$