Negative temperatures [tsl31]

Consider N noninteracting 2-level systems with energies $\pm\epsilon$.

$$N = N_+ + N_-, \quad U = (N_+ - N_-)\epsilon \quad \Rightarrow \; N_+ = \frac{1}{2}\left(N + \frac{U}{\epsilon}\right), \quad N_- = \frac{1}{2}\left(N - \frac{U}{\epsilon}\right).$$

Degeneracy of state with energy U: $N_U(U,N) = \dfrac{N!}{N_+!N_-!}$.

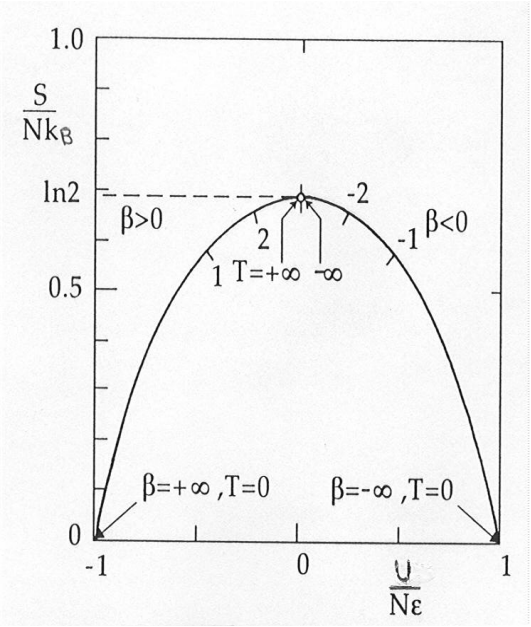
Entropy:

$$\begin{aligned} S(U,N) = k_B \ln N_U(U,N) = k_B N \ln N \quad &- \quad \frac{1}{2}k_B\left(N + \frac{U}{\epsilon}\right)\ln\left[\left(N + \frac{U}{\epsilon}\right)\right] \\ &- \quad \frac{1}{2}k_B\left(N - \frac{U}{\epsilon}\right)\ln\left[\left(N - \frac{U}{\epsilon}\right)\right]. \end{aligned}$$

Inverse temperature:

$$\frac{1}{T} = \left(\frac{\partial S}{\partial U}\right)_N = \frac{k_B}{2\epsilon}\ln\left(\frac{N - U/\epsilon}{N + U/\epsilon}\right).$$

Inversion of level occupancy corresponds to negative temperature.



[from Greiner et al. 1995]

Applications: laser pumping to metastable states, nuclear magnetism.