Microscopic states of ideal quantum gases [tln62]

Hamiltonian: $\hat{H}_N = \sum_{\ell=1}^{N} \hat{h}_\ell$.

1-particle eigenvalue equation: $\hat{h}_\ell |\mathbf{k}_\ell\rangle = \epsilon_\ell |\mathbf{k}_\ell\rangle$.

N-particle eigenvalue equation: $\hat{H}_N |\mathbf{k}_1, \ldots, \mathbf{k}_N\rangle = E_N |\mathbf{k}_1, \ldots, \mathbf{k}_N\rangle$.

Energy: $E_N = \sum_{\ell=1}^{N} \epsilon_\ell, \quad \epsilon_\ell = \dfrac{\hbar^2 \mathbf{k}_\ell^2}{2m}$.

N-particle product eigenstates: $|\mathbf{k}_1, \ldots, \mathbf{k}_N\rangle = |\mathbf{k}_1\rangle \ldots |\mathbf{k}_N\rangle$.

Symmetrized states for bosons: $|\mathbf{k}_1, \ldots, \mathbf{k}_N\rangle^{(S)}$.

- $N = 2$: $|\mathbf{k}_1, \mathbf{k}_2\rangle^{(S)} = \dfrac{1}{\sqrt{2}} \left(|\mathbf{k}_1\rangle|\mathbf{k}_2\rangle + |\mathbf{k}_2\rangle|\mathbf{k}_1\rangle\right)$.

Antisymmetrized states for fermions: $|\mathbf{k}_1, \ldots, \mathbf{k}_N\rangle^{(A)}$.

- $N = 2$: $|\mathbf{k}_1, \mathbf{k}_2\rangle^{(A)} = \dfrac{1}{\sqrt{2}} \left(|\mathbf{k}_1\rangle|\mathbf{k}_2\rangle - |\mathbf{k}_2\rangle|\mathbf{k}_1\rangle\right)$.

Occupation number representation: $|\mathbf{k}_1, \ldots, \mathbf{k}_N\rangle \equiv |n_1, n_2, \ldots\rangle$.

Here $\mathbf{k}_1$ represents the wave vector of the first particle, whereas n_1 refers to the number of particles in the first 1-particle state.

- energy: $\hat{H}|n_1, n_2, \ldots\rangle = E|n_1, n_2, \ldots\rangle, \quad E = \sum_{k=1}^{\infty} n_k \epsilon_k$.
- number of particles: $\hat{N}|n_1, n_2, \ldots\rangle = N|n_1, n_2, \ldots\rangle, \quad N = \sum_{k=1}^{\infty} n_k$.

ϵ_ℓ: energy of particle ℓ. $\quad$ ϵ_k: energy of 1-particle state k.

Allowed occupation numbers:

- bosons: $n_k = 0, 1, 2, \ldots$
- fermions: $n_k = 0, 1$.