Further ensemble averages [tln55]

Probability density in one-particle phase space:

$$\rho_l(\mathbf{q},\mathbf{p}) = \langle \delta(\mathbf{q}_l - \mathbf{q})\delta(\mathbf{p}_l - \mathbf{p})\rangle.$$

Position distribution and momentum distribution:

$$\rho_l(\mathbf{q}) = \langle \delta(\mathbf{q}_l - \mathbf{q})\rangle, \quad \rho_l(\mathbf{p}) = \langle \delta(\mathbf{p}_l - \mathbf{p})\rangle.$$

Distribution of distances and relative momenta between pairs of particles:

$$f_{lm}(r) = \langle \delta(r - |\mathbf{q}_l - \mathbf{q}_m|)\rangle, \quad F_{lm}(P) = \langle \delta(P - |\mathbf{p}_l - \mathbf{p}_m|)\rangle.$$

Average distance between pairs of particles:

$$\langle r_{lm}\rangle = \langle |\mathbf{q}_l - \mathbf{q}_m|\rangle = \int_0^\infty dr\, r f_{lm}(r).$$

Average magnitude of relative momentum between pairs of particles:

$$\langle P_{lm}\rangle = \langle |\mathbf{p}_l - \mathbf{p}_m|\rangle = \int_0^\infty dP\, P F_{lm}(P).$$

Applications to the classical ideal gas:

Noninteracting particles: $h(\mathbf{q},\mathbf{p}) = \dfrac{p^2}{2m} \;\Rightarrow\; \tilde{Z} = \dfrac{V}{\lambda_T^3}, \quad \lambda_T = \sqrt{\dfrac{h^2}{2\pi m k_B T}}.$

$\Rightarrow\;\; \rho_l(\mathbf{q},\mathbf{p}) = V^{-1}(2\pi m k_B T)^{-3/2} e^{-p^2/2mk_BT}.$

$\Rightarrow\;\; \rho_l(\mathbf{q}) = \displaystyle\int d^3p\, \rho_l(\mathbf{q},\mathbf{p}) = \frac{1}{V}.$

$\Rightarrow\;\; \rho_l(\mathbf{p}) = \displaystyle\int d^3q\, \rho_l(\mathbf{q},\mathbf{p}) = (2\pi m k_B T)^{-3/2} e^{-p^2/2mk_BT}.$

$$\rho_l(\mathbf{p})d^3p = f(\mathbf{v})d^3v \;\Rightarrow\; f(\mathbf{v}) = \left(\frac{m}{2\pi k_B T}\right)^{-3/2} e^{-mv^2/2k_BT}.$$

The spatial distribution of ideal gas particles in a uniform gravitational field (law of atmospheres) is calculated in exercise [tex79].

The distribution of relative momenta between pairs of ideal gas particles is calculated in exercise [tex80].