Systems of noninteracting particles [tln54]

Consider a classical systems of N noninteracting particles.

Hamiltonian: $H = \sum_{l=1}^{N} h_l(\mathbf{q}_l, \mathbf{p}_l)$.

Canonical partition function of distinguishable particles:

$$Z_N = \frac{1}{C_N}\int_\Gamma d^{6N}X\, e^{-\beta H(\mathbf{X})} = \prod_{l=1}^{N} \tilde{Z}_l, \quad \tilde{Z}_l = \frac{1}{h^3}\int d^3q_l\, d^3p_l\, e^{-\beta h_l(\mathbf{q}_l,\mathbf{p}_l)}.$$

Factorizing phase-space probability density:

$$\rho(\mathbf{X}) = \frac{1}{Z_N C_N}\, e^{-\beta H(\mathbf{X})} = \prod_{l=1}^{N}\left[\frac{1}{h^3\tilde{Z}_l} e^{-\beta h_l(\mathbf{q}_l,\mathbf{p}_l)}\right].$$

Identical one-particle Hamiltonians:

$$h_1 = \cdots = h_n \equiv h(\mathbf{q},\mathbf{p}) \quad \Rightarrow \; \tilde{Z}_1 = \cdots = \tilde{Z}_N = \tilde{Z} \equiv \frac{1}{h^3}\int d^3q\, d^3p\, e^{-\beta h(\mathbf{q},\mathbf{p})}.$$

$$\Rightarrow \; Z_N = \tilde{Z}^N, \quad \rho(\mathbf{X}) = \prod_{l=1}^{N}\left[\frac{1}{h^3\tilde{Z}} e^{-\beta h(\mathbf{q}_l,\mathbf{p}_l)}\right].$$

Indistinguishable particles:

$$Z_N = \frac{1}{N!}\tilde{Z}^N.$$

Note: It is important that we discriminate between noninteracting subsystems that are *identical but distinguishable* (e.g. atoms vibrating about rigid lattice sites) and noninteracting subsystems that are *identical and indistinguishable* (e.g. atoms of an ideal gas).