Fluctuations in a magnetic system [tln53]

Consider a system of N interacting magnetic moments m_i positioned in an external magnetic field of magnitude H.

Total magnetic moment: $M = \sum_{i=1}^{N} m_i$.

Hamiltonian: $\mathcal{H} = \mathcal{H}_{int} - HM$.

Canonical partition function: $Z_N = \mathrm{Tr}\, e^{-\beta\mathcal{H}}, \quad \beta = (k_B T)^{-1}$.

Gibbs free energy: $G(T, H, N) = -k_B T \ln Z_N$.

Magnetisation (average value of total magnetic moment):

$$\langle M\rangle = \frac{1}{Z_N}\mathrm{Tr}\left[M\, e^{-\beta\mathcal{H}}\right] = \beta^{-1}\frac{\partial}{\partial H}\ln Z_N \doteq M.$$

Enthalpy (average value of Hamiltonian):

$$\langle\mathcal{H}\rangle = \frac{1}{Z_N}\mathrm{Tr}\left[\mathcal{H}\, e^{-\beta\mathcal{H}}\right] = -\frac{\partial}{\partial\beta}\ln Z_N = U - HM = E$$

Energy fluctuations and heat capacity [tex109]:

$$\langle\mathcal{H}^2\rangle - \langle\mathcal{H}\rangle^2 = \frac{\partial^2}{\partial\beta^2}\ln Z_N = k_B T^2 C_H.$$

Magnetisation fluctuations and susceptibility [tex109]:

$$\langle M^2\rangle - \langle M\rangle^2 = \beta^{-2}\frac{\partial^2}{\partial H^2}\ln Z_N = k_B T\chi_T.$$