[mex91] Action-angle coordinates of the harmonic oscillator

Determine the canonical transformation $(q, p) \to (\theta, J)$ which produces the action-angle coordinates for the harmonic oscillator:

$$H(q,p) = \frac{p^2}{2m} + \frac{1}{2} m\omega_0^2 q^2 \quad \to \quad K(J).$$

(a) Find the transformed Hamiltonian $K(J)$. (b) Find the transformation relations $q(\theta, J),\ p(\theta, J)$. (c) Reconstruct the generating function $F_2(q, J)$. (d) Determine from F_2 the generating function $F_1(q, \theta)$ and verify that it is equal to function $F_1(q, Q)$ used in [mex86].

Solution: