## [mex90] Determine canonicity and generating functions II

Consider the following transformation from a set of canonical coordinates $(q, p)$ to a new set of coordinates $(Q, P)$:

$$Q = \frac{1}{2}(q^2 + p^2), \qquad P = -\arctan\frac{q}{p}.$$

(a) Verify that this transformation is canonical by investigating its Jacobian determinant. (b) Determine the generating function $F_4(p, P)$ by integration of the total differential $dF_4$. (c) Determine the generating function $F_1(q, Q)$ from $F_4(p, P)$ via Legendre transform.

**Solution:**