## [mex88] Particle with position-dependent mass moving in 1D potential

Consider a dynamical system with one degree of freedom specified by the equation of motion

$$\ddot{q} + G(q)\dot{q}^2 - F(q) = 0,$$

for arbitrary functions of $G(q)$ and $F(q)$. Show that any such system can be brought into canonical form, i.e. expressed as a pair of canonical equations by choosing the canonical momentum conjugate to $q$ as follows: $p = m(q)\dot{q}$, $m(q) \equiv \exp[2 \int dq\, G(q)]$. Express the associated Hamiltonian $H(q,p)$ in terms of the quantities $p$ (momentum), $m(q)$ (position-dependent mass) and $V(q) \equiv - \int dq\, F(q)m(q)$ (potential energy).

**Solution:**