[mex86] Canonical transformation applied to harmonic oscillator

Subject the Hamiltonian $H(q, p)$ of the harmonic oscillator to the canonical transformation specified by the generating function $F_1(q, Q)$:

$$H(q, p) = \frac{p^2}{2m} + \frac{1}{2}m\omega_0^2 q^2, \qquad F_1(q, Q) = \frac{1}{2}m\omega_0 q^2 \cot Q.$$

(a) Determine the transformation relations in the form $q = q(P, Q)$, $p = p(P, Q)$. (b) Determine the transformed Hamiltonian $K(Q, P)$. (c) Solve the canonical equations for $K(Q, P)$ to get the coordinates $Q(t), P(t)$. (d) Substitute these solutions into the transformation relations to obtain the time evolution of the original coordinates $q(t), p(t)$ for a given value of energy E.

Solution: