[mex85] Canonical transformation from rest frame to moving frame

Consider a particle of mass m moving along a straight line in a scalar potential. The Hamiltonian in the rest frame reads

$$H(q,p) = \frac{p^2}{2m} + V(q).$$

The function $F_2(q,P,t) = P[q - d(t)]$ generates a canonical transformation between the rest frame and a frame whose origin is displaced a distance $d(t)$ from the origin of the rest frame. (a) Determine the Hamiltonian $K(Q,P,t)$ after the transformation. (b) Compare the momenta p, P and the generalized velocities $\dot{q}, \dot{Q}$ of the particle in the two frames. (c) Determine the equations of motion in the form $m\ddot{q} = \ldots$ and $m\ddot{Q} = \ldots$ in the two frames of reference and explain the nature of all terms on the right-hand side of both equations.

Solution: