[mex79] Invariance under point transformations of Lagrange equations

Consider the point transformation $q_i = q_i(Q_1, \ldots, Q_n, t), i = 1, \ldots, n$ between two sets of generalized coordinates and the two functions $L(q_1, \ldots, q_n, \dot{q}_1, \ldots, \dot{q}_n, t) = \tilde{L}(Q_1, \ldots, Q_n, \dot{Q}_1, \ldots, \dot{Q}_n, t)$. Show by substitution of coordinates that if the q_i satisfy the Lagrange equations (1) then the Q_i satisfy the Lagrange equations (2):

$$\frac{\partial L}{\partial q_i} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} = 0, \quad i = 1, \ldots, n, \quad (1) \qquad \frac{\partial \tilde{L}}{\partial Q_i} - \frac{d}{dt}\frac{\partial \tilde{L}}{\partial \dot{Q}_i} = 0, \quad i = 1, \ldots, n. \quad (2)$$

Solution: