[mex77] Hamiltonian: conserved quantity or total energy?
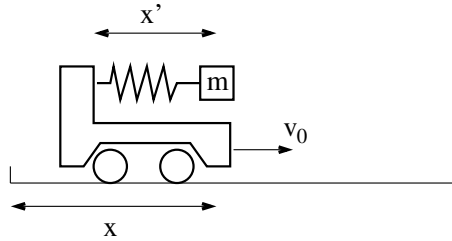
A harmonic oscillator (mass m, spring constant k) is attached to a cart that moves with constant velocity $\mathbf{v}_0$. Describe the dynamics in the coordinate system (x) that is at rest and in the coordinate system (x') that is moving with the cart.

(a) Construct the Lagrangian L of the oscillator in the rest frame and derive the associated Lagrange equation. Construct the Hamiltonian H from L.

(b) Construct the Lagrangian L' of the oscillator in the moving frame and derive the associated Lagrange equation. Construct the Hamiltonian H' from L'.

(c) Show that the Lagrange equations obtained in (a) and (b) are equivalent.

(d) Which of the two quantities H, H', if any, represents the total energy of the oscillator?

(e) Which of the two quantities H, H', if any, represents a conserved quantity?

Solution: