[mex67] Translational and rotational kinetic energies

A rigid body is regarded as composed of particles with mass m_α whose relative positions have constant magnitude. Consider the inertial coordinate system (X, Y, Z) and the coordinate system (x, y, z) with axes fixed to the rigid body. The velocity of particle α as expressed in the inertial frame is $\dot{\mathbf{R}} + \vec{\omega} \times \mathbf{r}_\alpha$, where $\mathbf{R}$ is the position of the origin of the body frame measured in the inertial frame, $\vec{\omega}$ is the instantaneous angular velocity of the body frame relative to the inertial frame, and $\mathbf{r}_\alpha$ is the position of particle α in the body frame. (a) Calculate the kinetic energy T of the rigid body in the inertial system. (b) Show that if the origin of the body frame is chosen at the center of mass, then the kinetic energy can be written as the sum of two terms where one represents the *translational* kinetic energy and the other the *rotational* kinetic energy.

Solution: