[mex6] Periodic motion in 2D phase space

Consider a particle of mass m with (conserved) energy E undergoing periodic motion with period τ in a potential $V(x)$. Let $A(E)$ be the area enclosed by the trajectory in 2D phase space $(x, m\dot{x})$. (a) Derive the following relation between period $\tau(E)$ and area $A(E)$:

$$\tau = \frac{dA}{dE}.$$

(b) Calculate the function $A(E)$ and derive from it the function $\tau(E)$ for the power-law potentials $V_2(x) = \frac{1}{2}kx^2$, $V_4(x) = \frac{1}{4}\alpha x^4$, and $V_1(x) = \beta|x|$.

Solution: