[mex47] Orbit of the inverse-square potential at small angular momentum

Consider the central force potential $V(r) = -\kappa/r^2$. If $\kappa > \ell^2/2m$, all orbits at $E > 0$ are unbounded and all orbits at $E < 0$ are bounded. (a) Show that these orbits can be expressed in the form

$$E > 0 : \ \frac{1}{r} = \sqrt{\frac{2mE}{2m\kappa - \ell^2}} \sinh\left(\vartheta \sqrt{\frac{2m\kappa}{\ell^2} - 1}\right), \quad E < 0 : \ \frac{1}{r} = \sqrt{\frac{2m|E|}{2m\kappa - \ell^2}} \cosh\left(\vartheta \sqrt{\frac{2m\kappa}{\ell^2} - 1}\right).$$

(b) Determine the time it takes the particle to move along the bounded orbit from r_{max} to the center of force ($r = 0$).

Solution: