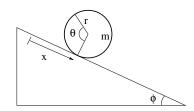## [mex32] Static frictional force of constraint

Consider a hoop of mass m and radius r rolling without slipping down an incline. (a) Determine the Lagrangian $L(x, \dot{x})$ of this one-degree-of-freedom system. Derive from it the Lagrange equation and its solution for initial condition $x_0 = 0, \dot{x}_0 = 0$. (b) Determine the alternative Lagrangian $L(x, \theta, \dot{x}, \dot{\theta})$ and the holonomic constraint $f(x, \theta) = 0$ that must accompany it. Derive the associated three equations of motion for the two unknown dynamical variables x, θ and the undetermined Lagrange multiplier λ. Solve these equations for the same initial conditions as in (a) and determine the static frictional force of constraint between the hoop and the incline.

Solution: