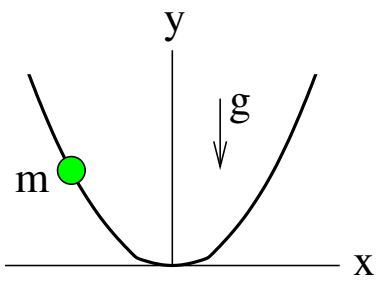
[mex276] Parabolic slide II

A bead of mass m slides without friction along a wire of parabolic shape, $y = Ax^2$, in a uniform gravitational field g pointing in the negative y-direction.
(a) Construct the Lagrangian $L(x, \dot{x})$.
(b) Derive the Lagrange equation for $x(t)$.
(c) Determine the angular frequency of small-amplitude oscillations.
(d) Construct the Hamiltonian $H(x, p_x)$ via Legendre transform and derive the canonical equations.
(e) Derive from the (alternative) Lagrangian $L(x, y, \dot{x}, \dot{y})$ and the equation of holonomic constraint, $f(x, y) = y - Ax^2 = 0$, three equations for the unknowns $x(t), y(t)$ (dynamical variables), and $\lambda(t)$ (Lagrange multiplier).
(f) Express the components (Q_x, Q_y) of the normal force (of constraint) acting on the bead during its motion in terms of $x, \dot{x}, \ddot{x}$.
(g) Interpret the result for the normal force at $x = 0$ as a combined reaction force to gravity and centrifugation.



Solution: