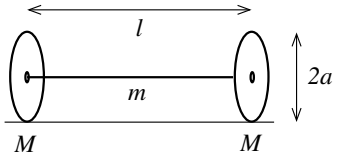
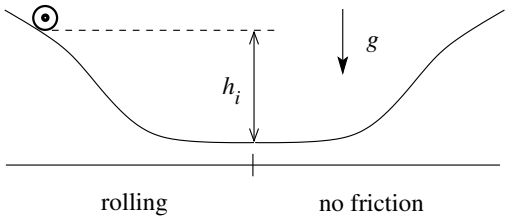
[mex272] Runaway dumbbell

Two solid disks, each of mass M and radius a, are mounted to the ends of a solid rod mass m, length l, and negligible width as shown.
(a) Find the principal moments of inertia of this dumbbell for rotations about its center of mass.
(b) When the dumbbell rolls from rest without sliping down a hill of altitude h_i in a uniform gravitational field g, what will be its final speed v? Assume that no mechanical energy is dissipated during the rolling motion.
(c) As the dumbbell continues its journey, now on a slippery surface up an adjacent hill, to what altitude will it climb before turning around? Assume the absence of any frictional force on this stretch.





Solution: