## [mex271] Linear central force potential

Consider a compact particle of mass m orbiting in the linear potential  $V(r) = \kappa r$ .

(a) Construct the Lagrangian  $L(r, \dot{r}, \dot{\theta})$  and infer from it the Routhian  $R(r, \dot{r}; \ell)$ .

(b) Find the angular momentum  $\ell$  and the energy E of a circular orbit with radius r.

(c) State Kepler's third law (relation between radius r and period  $\tau$ ) for circular orbits in this potential.

(d) A generic orbit of angular momentum,  $\ell$  and E will undergo radial motion between minimum radius  $r_{\rm P}$  (periapsis) and maximum radius  $r_{\rm A}$  (apsis). Establish a relation from which  $r_{\rm P}$  and  $r_{\rm A}$  can be determined for given E and  $\ell$ .

## Solution: