[mex269] Wiggling cylinder

A spring of stiffness k is mounted to a wall and to the axle of a solid cylinder. The cylinder has mass m and radius a. It is free to roll back and forth under the influence of the restoring force. The spring is relaxed when the cylinder touches the floor at x = 0.

- (a) Construct the Lagrangian $L(x, \dot{x})$.
- (b) Derive the Lagrange equation for the coordinate x.
- (c) Find the solution x(t) with initial conditions $x(0) = x_0$, $\dot{x}(0) = 0$.
- (d) Calculate the torque N(t) exerted by the floor on the cylinder.
- (e) Infer the Hamiltonian H(x, p) from $L(x, \dot{x})$ via Legendre transform.



Solution: