[mex262] Driven harmonic oscillator: runaway resonance

Consider the driven harmonic oscillator with no damping, $m\ddot{x} = -kx + F_0 \cos \omega t$. Take the general solution off resonance, $\omega \neq \omega_0 = \sqrt{k/m}$, and perform the limit $\omega \to \omega_0$ to show that the (runaway) solution at resonance with initial condition $x(0) = B \cos \beta$, $\dot{x}(0) = -\omega_0 B \sin \beta$ has the form

$$x(t) = B \cos(\omega_0 t + \beta) + \frac{F_0 t}{2m\omega_0} \sin(\omega_0 t).$$

Solution: