[mex260] Solid sphere rolling on plane

A solid sphere of mass m and radius a is rolling without slipping on the xy-plane under the influence of an external force $\mathbf{F} = (F_x, F_y, F_z)$ and an external torque $\mathbf{N} = (N_x, N_y, N_z)$, both acting on its center of mass. The rolling motion is described by the instantaneous velocity $\mathbf{V} = (V_x, V_y, V_z)$ of the center of mass and the instantaneous angular velocity $\vec{\omega} = (\omega_x, \omega_y, \omega_z)$ about its center of mass. In [mln106] we have established the equations of motion,

$$m\frac{d\mathbf{V}}{dt} = \mathbf{F} + \mathbf{F}^{\mathrm{c}}, \quad I\frac{d\vec{\omega}}{dt} = \mathbf{N} - a\hat{\mathbf{n}} \times \mathbf{F}^{\mathrm{c}},$$

and the equation of constraint,

$$\dot{\mathbf{V}} = a\,\dot{\vec{\omega}} \times \hat{\mathbf{n}}$$

Eliminate the contact force of constraint, $\mathbf{F}^{\mathrm{c}}$, from these relations to arrive at the equations of motion for $\mathbf{V}$ and $\vec{\omega}$ reduced to quadrature as stated in [mln106].

Solution: