## [mex244] Classical inverse scattering problem II

The reconstruction of the (central force) scattering potential $V(r)$ from the observed scattering cross section $\sigma(\theta)$ as outlined in [mln104] involves the conversion of the relation

$$\pi\sqrt{\alpha} - \int_0^\alpha dx\,\tilde{\theta}'(x)\sqrt{\alpha - x} = \pi\int_0^{u_m(\alpha)} \frac{du}{w(u)}, \qquad u_m^2 = \alpha[w(u_m)]^2,$$

as derived in [mex243] into the differential relation

$$\pi\frac{dw}{w} = -d\left(\frac{u}{w}\right)\int_0^{u^2/w^2} dx \frac{\tilde{\theta}'(x)}{\sqrt{u^2/w^2 - x}}$$

by setting $\alpha = u^2/w^2$, taking the derivative with respect to $u$on both sides and multiplying back by $du$. Show that the boundary value $u_m(\alpha)$ becomes the unrestricted $u$ in the process.

**Solution:**