[mex241] Elastic collision: velocities of scattered particles

A particle of mass m_1 and incident velocity $\bar{\mathbf{v}}_0$ undergoes an elastic collision via central force with a particle of mass m_2 that is initially at rest. Show that the velocities of the scattered particles depend on the scattering angles in the laboratory frmae as follows:

$$\frac{\bar{v}_2}{\bar{v}_0} = 2\frac{m}{m_2}\cos\bar{\theta}_2, \quad m \doteq \frac{m_1 m_2}{m_1 + m_2},$$

$$\frac{\bar{v}_1}{\bar{v}_0} = \frac{m}{m_2}\left(\cos\bar{\theta}_1 \pm \sqrt{\frac{m_2^2}{m_1^2} - \sin^2\bar{\theta}_1}\right),$$

where two solutions ($\pm$) exist for $m_1 > m_2$ and one solution ($+$) for $m_1 < m_2$.

Solution: