[mex22] Find a simpler Lagrangian

Consider a dynamical system with one degree of freedom specified by the Lagrangian

$$L = \frac{1}{12}m^2\dot{x}^4 + m\dot{x}^2W(x) - W^2(x),$$

where m is the mass of a particle that can move along the x-axis and W(x) is a differentiable function. Determine the equation of motion for the dynamical variable x and explain the role of the function W(x). Find a simpler Lagrangian that leads to the same equation of motion.

Solution: