[mex2] Gravitational collapse of cold cloud of dust

Imagine a spherically symmetric, homogeneous cloud of dust grains. All grains are initially at rest and separated from one another by distances large compared to the grain size. Initially the cloud has radius R_0 and average mass density ρ_0 . Assume that collisions between grains can be ignored, which would cause a pressure to build up.

(a) Calculate the time T it takes for the cloud to collapse to (essentially) zero radius. Note that T is independent of R_0 for given ρ_0 .

(b) Show that at all times during the contraction the speed of each dust grain is proportional to its distance from the center of the cloud as in Hubble's law.

Solution: