[mex199] Exponential potential

Consider a particle of mass $m = 1/2$ moving in a straight line (x-axis) and subject to a force $F(x) = -e^x$. Find the solution $x(t)$, $p(t)$ as follows:

(a) Find a generating function $F_2(x, P)$ which transforms the Hamiltonian $H(x, p) = p^2 + e^x$ into $K(Q, P) = \frac{1}{4}P^2$ and derive canonical transformation relations $Q(x, p)$ and $P(x, p)$ from $F_2(x, P)$.

(b) Solve the canonical equations for $K(Q, P)$ to get $Q(t)$ and $P(t)$ and substitute these solutions into the inverse transformation relations $x(Q, P) = x(t)$ and $p(Q, P) = p(t)$.

(c) State the solutions $x(t), p(t)$ for initial conditions $x(0) = p(0) = 0$. Verify that $x(t)$ and $p(t)$ thus found are indeed solutions of the canonical equations for $H(x, p)$.

Solution: