[mex196] Electromagnetic gauge transformation

Consider a particle of mass m and electric charge q moving (nonrelativistically) in an electromagnetic field described by the vector potential $\mathbf{A}(\mathbf{r},t)$ and the scalar potential $\phi(\mathbf{r},t)$. Show that the gauge transformation

$$\mathbf{A} \to \mathbf{A}' \doteq \mathbf{A} + \nabla f(\mathbf{r},t), \qquad \phi \to \phi' \doteq \phi - \frac{1}{c}\frac{\partial}{\partial t} f(\mathbf{r},t)$$

is a canonical transformation. Find the generating function $F_2(\mathbf{r},\mathbf{P},t)$.

Solution: