[mex195] Canonicity of gauge transformation

Consider the gauge transformation $L(q, \dot{q}, t) \to \tilde{L}(q, \dot{q}, t)$ with

$$\tilde{L}(q, \dot{q}, t) = L(q, \dot{q}, t) + \frac{d}{dt} f(q, t),$$

which we have shown in [mex21] to leave the Lagrange equations invariant. Show that this transformation is canonical and find its generating function $F_2(q, P, t)$. Find also the gauge-transformed Hamiltonian $\tilde{H}$.

Solution: