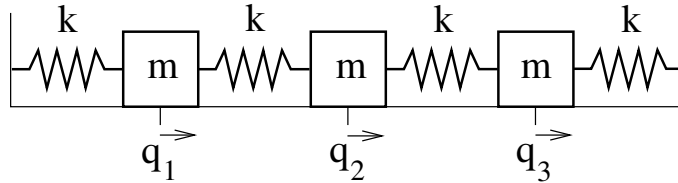## [mex187] Three coupled oscillators

Consider the elastic chain consisting of three blocks and four springs as shown. (a) Show that the equations of motion for the generalized coordinates q_j can be brought into the form

$$m\ddot{q}_j + k(2q_j - q_{j-1} - q_{j+1}) = 0, \quad j = 1, 2, 3$$

with boundary conditions $q_0(t) = q_4(t) = 0$. (b) Use the ansatz $q_j(t) = A_j \cos(\omega t)$ and find the three normal-mode frequencies $\omega_r, r = 1, 2, 3$. (c) Find the normal coordinates $Q_j = \sum_i A_{ij} q_i, j = 1, 2, 3$. (d) Illustrate each normal mode Q_j modes by plotting $q_i(0)$ versus i.

Solution: