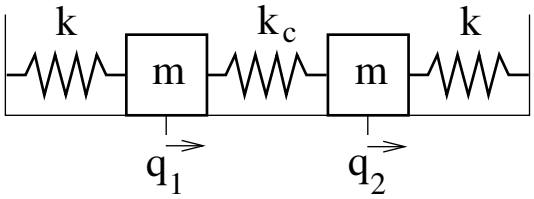
[mex186] Two coupled oscillators

Consider a system of two blocks of mass m attached by springs of stiffness k to rigid walls on two sides and by a spring of stiffness k_c to each other. The blocks can slide back and forth without friction. (a) Find the Lagrangian $L(q_1, q_2, \dot{q}_1, \dot{q}_2)$. (b) Write the equations of motion for q_1, q_2. (c) Find the normal mode frequencies ω_1, ω_2 by solving the characteristic equation. (d) Rewrite the Lagrangian in the form $L = \frac{1}{2}\sum_{ij}[m_{ij}\dot{q}_i\dot{q}_j + k_{ij}q_iq_j]$. Then find the orthogonal matrix A_{ij} which diagonalizes the symmetric matrices m_{ij} and k_{ij} simultaneously: $\sum_{lm} A^T_{il} m_{lm} A_{mj} = \delta_{ij}$, $\sum_{lm} A^T_{il} k_{lm} A_{mj} = \omega_i \delta_{ij}$. (e) Find the normal mode coordinates $Q_j = \sum_i A_{ij} q_i$.



Solution: