[mex169] Elliptic and hyperbolic orbits

Calculate the orbital integral (a) for an orbit with energy $E > 0$ and angular momentum ℓ of the attractive central-force potential $V(r) = \frac{1}{2}kr^2$ and (b) for an orbit with energy $E < 0$ and angular momentum ℓ of the repulsive central-force potential $V(r) = -\frac{1}{2}kr^2$. Show that the solutions (a) and (b) can be cast into the form $x^2/a^2 \pm y^2/b^2 = 1$, respectively, if the Cartesian axes are suitably oriented. Find the parameters a and b in each case as functions of E, ℓ, m, k. Express E and ℓ in terms of a, b, m, k in each case.

Solution: