[mex147] Plane pendulum IV: separatrix motion and rotations

The plane pendulum consists of a point mass m constrained by a massless rod to move in a vertical circle of radius ℓ in a uniform gravitational field g.
(a) By reduction to quadrature find the angle $\theta(t)$ of the separatrix motion (at energy $E = 2mg\ell$).
(b) By the same method find the angle $\theta(t)$ of the rotational motion (at energy $E > 2mg\ell$). Establish the familiar result of uniform rotation at very high energies ($E \gg 2mg\ell$)
(c) Find the period of rotation T as a function of energy. Expand the exact expression $T(E)$ at high energies to include the leading correction due to gravity.

Solution: