[mex145] Close encounter of the first kind

A rock of mass m approaches the solar system with a velocity v_0, and if it had not been attracted toward the sun it would have missed the sun by a distance d. Neglect the influence of the planets. Show that the closest approach of the orbit is

$$a = \sqrt{d^2 + d_0^2} - d_0, \quad d_0 \doteq \frac{Gm_\odot}{v_0^2}.$$

Solution: