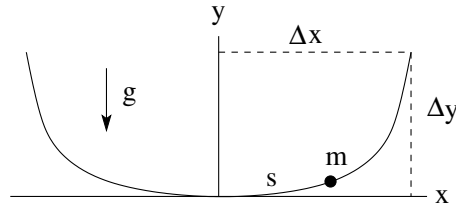## [mex144] Isochronous potential well

A particle is constrained to move under the influence of a uniform gravitational field g on a curve $y(x)$ with a minimum at $x = 0$ in a vertical plane. Find the shape of the curve such that the oscillations of the particle about this potential minimum have a period that is independent of the amplitude. This is accomplished by requiring that the potential energy (here mgy) is proportional to the square of the arc length s from $x = 0$, just as is the case in a harmonic oscillator. Use $\frac{1}{2}ks^2 = mgy$, where k is the equivalent spring stiffness, and set $mg/k = 4a$, where a is a characteristic length of the potential well. Find the maximum half width Δx and the maximum height Δy of the potential well for which this scheme works and express these measures in units of a. Find the value of a which makes the period of oscillation one second (1s) for $g = 9.8 \mathrm{m/s^2}$.

Solution: