[mex13] Predator and prey

A population F of foxes feeds on a population H of hares. The birth rate of foxes is proportional to the fox population and to the amount of food available. Foxes die at a rate proportional to the fox population. Hares die primarily through encounters with foxes and are born at a rate proportional to the hare population:

$$\dot{H} = aH - bHF, \qquad \dot{F} = cHF - dF,$$

where a, b, c, d are positive constants and $H \geq 0, F \geq 0$ is assumed.
(a) Find all fixed points in the (H, F)-plane and determine their nature. Sketch the phase portrait and give an interpretation of the phase flow.
(b) If the population of hares is suddenly decimated by an epidemic disease from which the remaining hares are immune, discuss the different effects this can have on the system depending on the size of the fox population at the time the hare population is reduced by the disease.

Solution: