[mex126] Angle between apsidal vectors for nearly circular orbits

Consider a particle of mass m and angular momentum ℓ subject to a central force F(r) = -V'(r)and moving in a stable circular orbit of radius r = R. Show that nearly circular orbits in the immediate vicinity have an apsidal angle

$$\Delta \vartheta = \pi \sqrt{\frac{V'(R)}{3V'(R) + RV''(R)}}.$$

Solution: