[mex125] Small oscillations of radial coordinate about circular orbit

Consider a particle of mass m and angular momentum ℓ subject to a central force $F(r) = -V'(r)$. Under the conditions stated in [mex53] that a stable orbit at radius $r = R$ exists, show that on an orbit starting at radius $r = R + x$ with $|x| \ll R$ next to a stable circular orbit of radius R, the radial coordinate oscillates about R with angular frequency $\omega_0^2 = -3F(R)/mR - F'(R)/m$.

Solution: