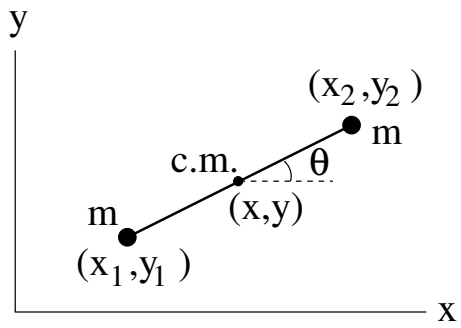
[mex122] Massive dimer on skates

Consider two particles of mass m connected by a rigid rod of negligible mass and length ℓ which rotates freely about its center of mass. In addition, the center of mass undergoes translational motion constrained by the requirement that it must by perpendicular to the direction of the rod at all times. This system has one holonomic and one nonholonomic constraint. (a) Use the holonomic constraint to express the Lagrangian $L(x, y, \theta, \dot{x}, \dot{y}, \dot{\theta})$, where x, y are the center-of-mass coordinates and the θ is the angle between the rod and the x-axis. (b) Express the nonholonomic constraint as a relation between $\dot{x}, \dot{y}, \theta$. (c) Derive the equations of motion in the form of three Lagrange equations and one equation of (nonholonomic) constraint. (d) Solve these equations of motion for the initial conditions $x(0) = y(0) = 0$, $\dot{x}(0) = 0$, $\dot{y}(0) = v_0 > 0$, $\theta(0) = 0$, $\dot{\theta}(0) = \omega > 0$. (e) Determine the forces of nonholonomic constraint.



Solution: