

[mex12] Fixed points of the plane pendulum

Consider the equation of motion

$$\ddot{\theta} + 2\beta\dot{\theta} + \omega_0^2 \sin \theta = 0,$$

where $\omega_0 = \sqrt{g/L}$ is the characteristic frequency and β is the damping parameter.

(a) Determine the nature of the two fixed points for (i) zero damping ($\beta = 0$), (ii) weak damping ($\beta < \omega_0$), (iii) critical damping ($\beta = \omega_0$), and (iv) strong damping ($\beta > \omega_0$).

(b) Use the Mathematica `StreamPlot` command or equivalent to graphically present the phase flow near the fixed point at associated with $\theta = 0$ for the cases (i)-(iv). Adjust the style and range of your graph such that the differences between the four cases are optimally visible.

Solution: