[mex108] Isoclines and fixed points

Two species of animals vie for the same food source. In isolation each species grows logistically. Through interaction they impede each other's growth. The equations of motion for the two populations N_1, N_2,

$$\dot{N}_1 = rN_1\left(1 - \frac{N_1}{K}\right) - \alpha N_1 N_2, \qquad \dot{N}_2 = sN_2\left(1 - \frac{N_2}{L}\right) - \beta N_1 N_2,$$

depend on six paramters: r, s are the per-capita growth rates, K, L the carrying capacities, for the two populations N_1, N_2, respectively, and α, β are the adverse impact parameters of the competitor population. Consider the two cases (i) $K = L = 2, r = s = 1, \alpha = \beta = 1$, and (ii) $K = L = 1, r = s = 2, \alpha = \beta = 1$.

For each case determine all curves of vertical and horizontal isoclines. Draw all these lines in a diagram N_1 versus N_2 for each case. Indicate the location of all four fixed points in each case as the intersection points between curves belonging to the vertical and horizontal isloclines. Determine the coordinates of all fixed points in the (N_1, N_2)-plane.

Solution: