One Degree of Freedom [mln71]

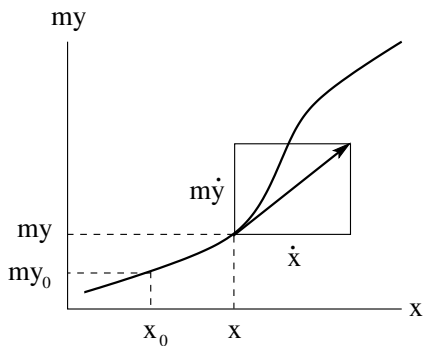
Newton's equation of motion (for autonomous system):

$$m\ddot{x} = F(x, \dot{x}). \tag{1}$$

- Conservative system: $F = F(x)$ $\Rightarrow$ solution by quadrature [mln4].
- Non-autononmous system: $F = F(x, \dot{x}, t)$ $\Rightarrow$ transformation to autonomous system with two degrees of freedom.

Equivalent (phase-plane) representation of (1):

$$\dot{x} = y, \qquad m\dot{y} = F(x, y). \tag{2}$$



- Equations (2) determine a vector field in phase plane (x, my).
- Any solution of (2) for given initial condition (x_0, my_0) describes a trajectory in phase plane.
- Trajectories are tangential to vector field $(\dot{x}, m\dot{y})$.
- Phase plane is filled with trajectories (phase flow).
- Trajectories do not intersect (Cauchy's existence theorem).
- At fixed points $(\dot{x} = 0, m\dot{y} = 0)$ system is at rest.
- In conservative systems, all trajectories lie on lines of constant energy, $E(x, y) = \text{const}$.
- Phase portrait describes salient features of phase flow [msl5].
- Closed trajectories describe periodic motion.
- In conservative systems the phase flow is incompressible.