Mass and Energy [mln62]

Consider an autonomous system (internal forces only).

- The postulates of Newtonian mechanics imply separate conservation laws for the total momentum and the total mass.
- The postulates of relativistic mechanics are incompatible with separate conservation laws for mass and momentum.

Einstein's thought experiment:

Consider a macroscopic rectangular box of mass M and length L at rest in the inertial frame of the observer. A light pulse of momentum p and energy $E = pc$ is emitted at one end and absorbed at the other end.

According to Newtonian mechanics, energy is transferred from one end to the other end of the box, but no mass is transferred.

The box recoils a distance Δx during the time of flight Δt of the light pulse in accordance with momentum conservation.

$$Mv = -\frac{E}{c}, \quad \Delta x = v\Delta t, \quad \Delta t = \frac{L}{c} \quad \Rightarrow \; \Delta x = -\frac{EL}{Mc^2}.$$

Problem: the nonzero displacement Δx of the box is incompatible with zero mass transfer. To reconcile momentum conservation with zero shift of the center of mass of the box, we must attribute a mass m to the transferred energy:

$$\Delta x_{\rm cm} = 0 \quad \Rightarrow \; mL + M\Delta x = 0 \quad \Rightarrow \; m = \frac{E}{c^2} \quad \Rightarrow \; E = mc^2.$$

Correct for shortened distance traveled by light and for mass loss of box:

$$m(L + \Delta x) + (M - m)\Delta x = 0 \quad \Rightarrow \; E = mc^2 \qquad \text{(no change!)}$$