Skater's Paradox [mln61]

A skater with blades of proper length $\ell_0 = 15\text{in}$ on his skates moves with velocity $v = 0.8c$ relative to a flat ice surface, approaching a gap of proper width $d_0 = 10\text{in}$.

Skater's perspective (frame S'):

The gap in the ice is Lorentz contracted to a width $d = d_0\sqrt{1-(0.8)^2} = 6\text{in}$, which is shorter than the length $\ell_0 = 15\text{in}$ of his blades. Therefore, the front end of the blade will gain support on the far side of the gap before the back end loses support on the near side. The skater concludes that he will make it across the gap without accident.

Spectator's perspective (frame S):

The blades are Lorentz contracted to length $\ell = \ell_0\sqrt{1-(0.8)^2} = 9\text{in}$, which is shorter than the length $d_0 = 10\text{in}$ of the gap in the ice. Therefore, the back end of the blade loses support on the near side of the gap before the front end is able to gain support on the far side. The spectator concludes that the skater will not make it across the gap without accident.

Analysis:

Event 1: Back end of blade enters gap.
Event 2: Front end of blade exits gap.

Check Lorentz invariant quantity $(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2$ between events.

- Frame S': $x'_1 = 0, \quad x'_2 = 15\text{in}, \quad t'_2 = 0, \quad t'_1 = (9\text{in})/v.$
  $\Rightarrow \ (\Delta s)^2 = (11.25\text{in})^2 - (15\text{in})^2 = -(98.4375\text{in})^2.$
- Frame S: $x_1 = 0, \quad x_2 = 10\text{in}, \quad t_1 = 0, \quad t_2 = (1\text{in})/v.$
  $\Rightarrow \ (\Delta s)^2 = (1.25\text{in})^2 - (10\text{in})^2 = -(98.4375\text{in})^2.$

Conclusion:

Events 1 and 2 have a space-like relationship. They have no definite time-ordering as demonstrated. Such events cannot be causally related.

The skater implies a causal relation, which is fallacious. He treats the blade as a rigid body, transporting information across the gap faster than the speed of light. He sinks into the gap and falls. $\Rightarrow$ [mex217].