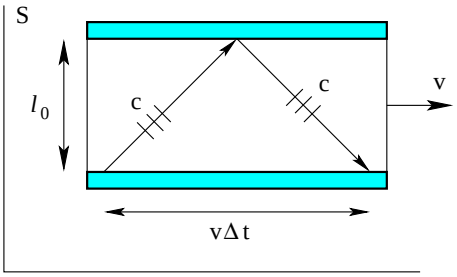
Relativity of Space and Time [mln50]

Frame S' moves with velocity v relative to frame S.
Clock is at rest in frame S'.
Distance traveled by signal in frame S': $2\ell_0$ (proper length ℓ_0).
Time period measured in frame S': $\Delta\tau = 2\ell_0/c$ (proper time).

Time dilation

Distance traveled by signal in S: $2\sqrt{\ell_0^2 + (v\Delta t/2)^2} = c\Delta t$.

Time period measured in S: $\Delta t = \dfrac{2\ell_0/c}{\sqrt{1 - v^2/c^2}} = \dfrac{\Delta\tau}{\sqrt{1 - v^2/c^2}}$.



Length contraction

Distance traveled by signal in S: $c(\Delta t_1 + \Delta t_2) = (\ell + v\Delta t_1) + (\ell - v\Delta t_2)$.

Time period measured in S: $\Delta t = \Delta t_1 + \Delta t_2 = \dfrac{\ell}{c - v} + \dfrac{\ell}{c + v} = \dfrac{2\ell/c}{1 - v^2/c^2}$.

Comparison with proper time and length: $\Delta t = \dfrac{\Delta\tau}{\sqrt{1 - v^2/c^2}} = \dfrac{2\ell_0/c}{\sqrt{1 - v^2/c^2}}$.

Length contraction: $\ell = \ell_0\sqrt{1 - v^2/c^2}$.