Electric Potential of a Uniformly Charged Solid Sphere

- Electric charge on sphere: $Q = \rho V = \dfrac{4\pi}{3}\rho R^3$

- Electric field at $r > R$: $E = \dfrac{kQ}{r^2}$

- Electric field at $r < R$: $E = \dfrac{kQ}{R^3}\, r$

- Electric potential at $r > R$:

$$V = -\int_{\infty}^{r} \frac{kQ}{r^2}\, dr = \frac{kQ}{r}$$

- Electric potential at $r < R$:

$$V = -\int_{\infty}^{R} \frac{kQ}{r^2}\, dr - \int_{R}^{r} \frac{kQ}{R^3}\, r\, dr$$

$$\Rightarrow V = \frac{kQ}{R} - \frac{kQ}{2R^3}\left(r^2 - R^2\right) = \frac{kQ}{2R}\left(3 - \frac{r^2}{R^2}\right)$$

$V = \dfrac{kQ}{2R}\left(3 - \dfrac{r^2}{R^2}\right)$

$V = \dfrac{kQ}{r}$