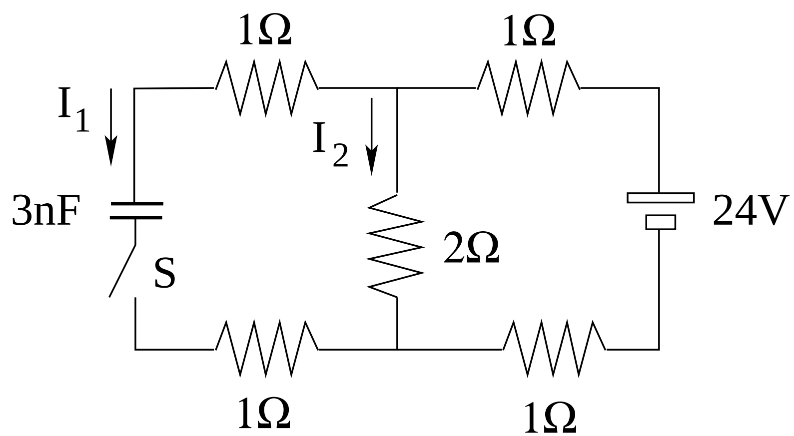
Unit Exam III: Problem #1 (Spring ’19)

This circuit is in a steady state with the switch open and the capacitor discharged.
(a) Find the currents I_1 and I_2 while the switch is still open.
(b) Find the currents I_1 and I_2 right after the switch has been closed.
(c) Find the currents I_1 and I_2 a long time later.
(d) Find the voltage V across the capacitor, also a long time later.



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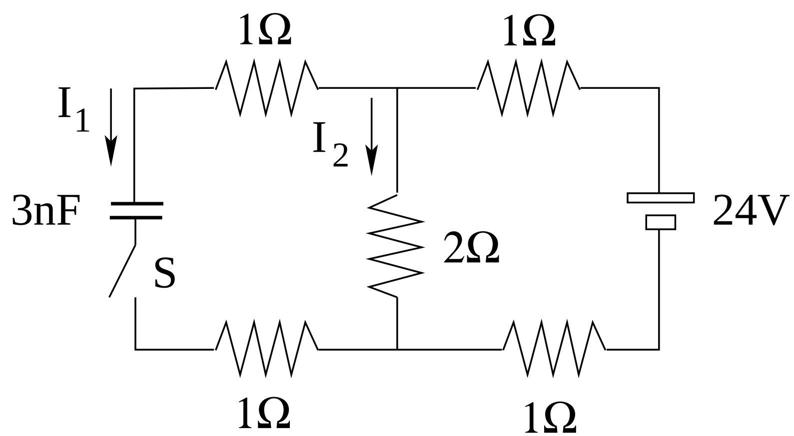


Solution:

(a) $I_1 = 0, \quad I_2 = \dfrac{24\text{V}}{1\Omega + 2\Omega + 1\Omega} = 6\text{A}.$

(b) $R_{eq} = 1\Omega + \left(\dfrac{1}{2\Omega} + \dfrac{1}{1\Omega + 1\Omega}\right)^{-1} + 1\Omega = 3\Omega$ (capacitor discharged)

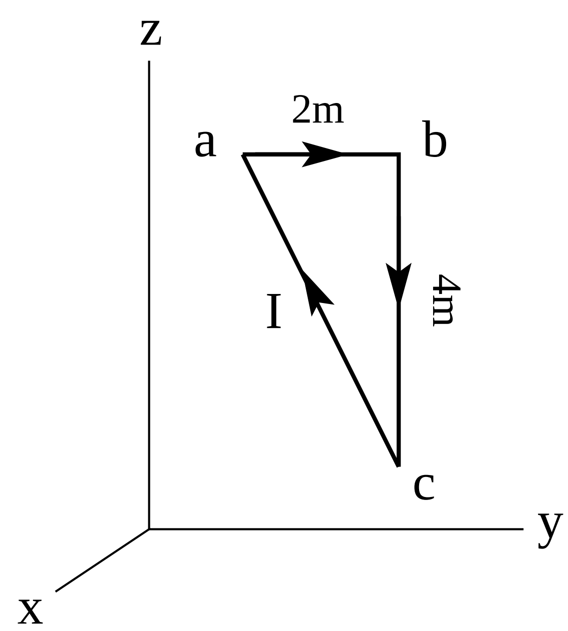
$$\Rightarrow I_1 + I_2 = \frac{24\text{V}}{3\Omega} = 8\text{A}, \quad I_1 = I_2 = 4\text{A}.$$

(c) capacitor fully charged: $I_1 = 0, \quad I_2 = \dfrac{24\text{V}}{1\Omega + 2\Omega + 1\Omega} = 6\text{A}.$

(d) loop rule: $(2\Omega)(6\text{A}) - (1\Omega)(0\text{A}) - V - (1\Omega)(0\text{A}) = 0 \quad \Rightarrow \quad V = 12V.$

Unit Exam III: Problem #2 (Spring '19)

Consider a region with uniform magnetic field $\vec{B} = 3\mathrm{T}\hat{\mathbf{j}} + 5\mathrm{T}\hat{\mathbf{k}}$. A conducting loop positioned in the yz-plane has the shape of a right-angled triangle and carries a clockwise current $I = 2$A.
(a) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the loop.
(b) Find the force $\vec{F}_{ab}$ (magnitude and direction) acting on side ab.
(c) Find the force $\vec{F}_{bc}$ (magnitude and direction) acting on side bc.
(d) Find the torque $\vec{\tau}$ (magnitude and direction) acting on the loop.



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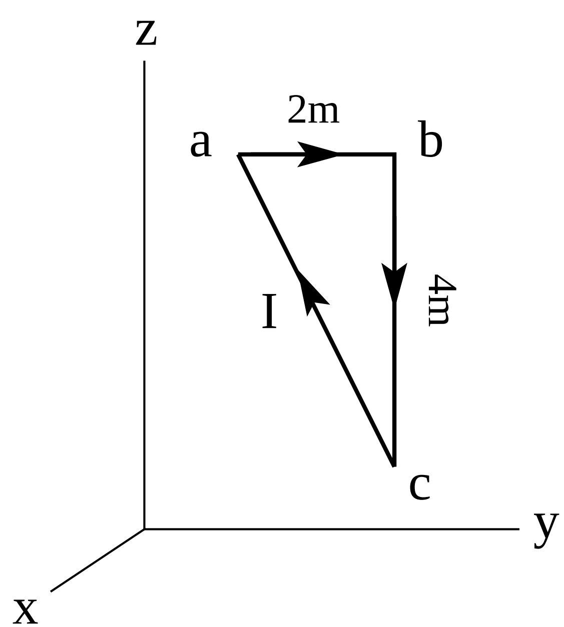


Solution:

(a) $\vec{\mu} = -(2\text{A})(4\text{m}^2)\hat{\mathbf{i}} = -8\text{Am}^2\hat{\mathbf{i}}$.

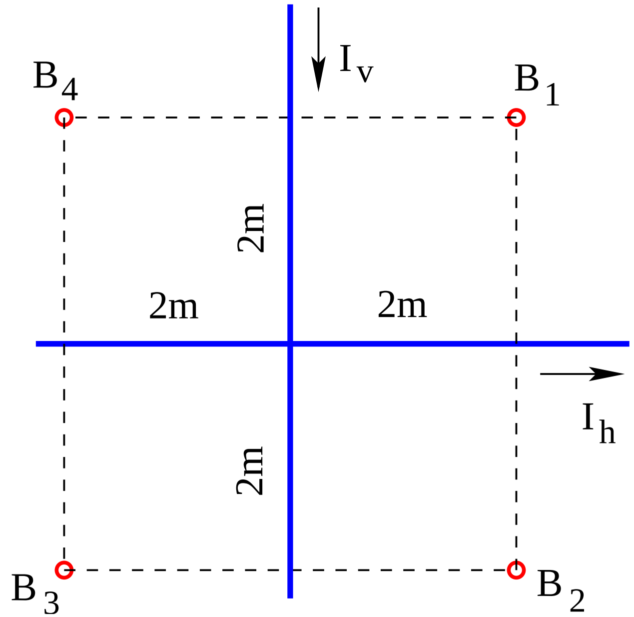
(b) $\vec{F}_{ab} = (2\text{A})(2\text{m}\hat{\mathbf{j}}) \times \left[3\text{T}\hat{\mathbf{j}} + 5\text{T}\hat{\mathbf{k}}\right] = 20\text{N}\hat{\mathbf{i}}$.

(c) $\vec{F}_{bc} = (2\text{A})(-4\text{m}\hat{\mathbf{k}}) \times \left[3\text{T}\hat{\mathbf{j}} + 5\text{T}\hat{\mathbf{k}}\right] = 24\text{N}\hat{\mathbf{i}}$.

(d) $\vec{\tau} = (-8\text{Am}^2\hat{\mathbf{i}}) \times \left[3\text{T}\hat{\mathbf{j}} + 5\text{T}\hat{\mathbf{k}}\right] = -24\text{Nm}\hat{\mathbf{k}} + 40\text{Nm}\hat{\mathbf{j}}$

Unit Exam III: Problem #3 (Spring ’19)

Consider two infinitely long, straight wires with currents $I_v = 3$A, $I_h = 3$A in the directions shown. Find direction (in/out) and magnitude of the magnetic fields $\mathbf{B}_1$, $\mathbf{B}_2$, $\mathbf{B}_3$, $\mathbf{B}_4$, at the points marked in the graph.



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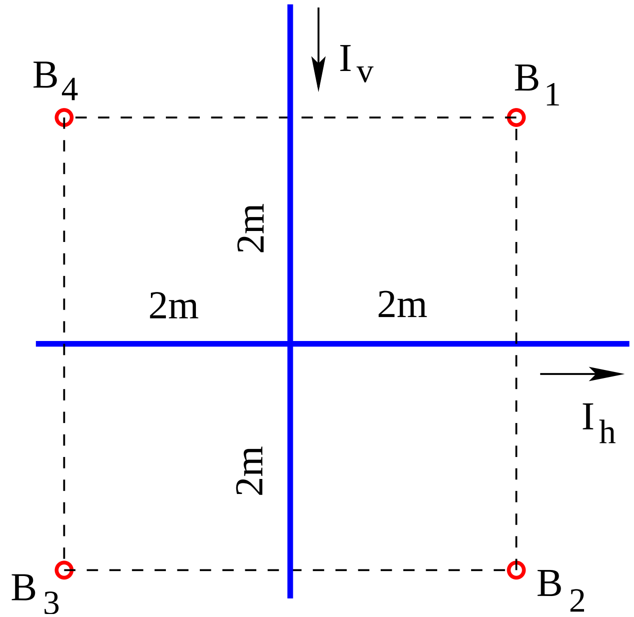

Solution:

- $B_1 = \frac{\mu_0}{2\pi}\left(\frac{I_v}{2\text{m}} + \frac{I_h}{2\text{m}}\right) = +6 \times 10^{-7}\text{T}$ (out).
- $B_2 = \frac{\mu_0}{2\pi}\left(\frac{I_v}{2\text{m}} - \frac{I_h}{2\text{m}}\right) = 0.$
- $B_3 = \frac{\mu_0}{2\pi}\left(-\frac{I_v}{2\text{m}} - \frac{I_h}{2\text{m}}\right) = -6 \times 10^{-7}\text{T}$ (in).
- $B_4 = \frac{\mu_0}{2\pi}\left(-\frac{I_v}{2\text{m}} + \frac{I_h}{2\text{m}}\right) = 0.$